Lezione 2
Soluzioni Esercizi

Sol. Ex. 2.1.

- $\left(\frac{4}{5}\right)^2 = \left(\frac{4}{5}\right)\left(\frac{4}{5}\right) = \frac{16}{25}.$
- $6 \cdot 6 \cdot 6 = 216 \quad \text{oppure} \quad 6^3 = (2\cdot 3)^3 = 2^3 \cdot 3^3 = 8 \cdot 27 = 216.$

Sol. Ex. 2.2.

- $(-5)^2 = (-5)(-5) = 25.$
- $-3^2 = -(3\cdot 3) = -9.$
- $\left(-\frac{2}{5}\right)^3 = \left(-\frac{2}{5}\right)\cdot\left(-\frac{2}{5}\right)\cdot\left(-\frac{2}{5}\right) = -\frac{8}{125}.$
- $-2^3 = -(2\cdot 2\cdot 2) = -8 = (-2)^3.$

Sol. Ex. 2.3.

- $3 > 1 \quad , \quad 5 < 8 \quad \text{quindi} \quad 3^5 < 3^8.$
- $0 < \frac{1}{6} < 1 \quad , \quad 4 < 7 \quad \text{quindi} \quad \left(\frac{1}{6}\right)^7 < \left(\frac{1}{6}\right)^4.$
- $\left(-\frac{1}{3}\right)^3 < 0 \quad , \quad \left(-\frac{1}{3}\right)^2 > 0 \quad \text{quindi} \quad \left(-\frac{1}{3}\right)^3 < \left(-\frac{1}{3}\right)^2.$

Sol. Ex. 2.4.

- $0 < \frac{2}{3} < \frac{3}{4}$ e l'esponente è lo stesso, quindi $\left(\frac{2}{3}\right)^5 < \left(\frac{3}{4}\right)^5.$
- $\left(-\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^4$ (l'esponente è pari).
- $-3^4 < 0 \quad , \quad 2^4 > 0 \quad \text{quindi} \quad -3^4 < 2^4.$

Sol. Ex. 2.5.

- $\left(\frac{7}{5}\right)^2\left(-\frac{8}{5}\right)^{-2} = \left(\frac{7}{5}\right)^2\left(-\frac{5}{8}\right)^2 = \left(\frac{7}{5}\cdot\frac{5}{8}\right)^2 = \left(\frac{7}{8}\right)^2 = \frac{49}{64}.$
- $\left(\frac{7}{3}\right)^{-2} : \left(\frac{6}{7}\right)^3 = \left(\frac{3}{7}\right)^2 : \left(\frac{6}{7}\right)^3 = \left(\frac{3}{7}\right)^2 \cdot \left(\frac{7}{6}\right)^3 = \frac{3^2}{7^2}\cdot\frac{7^3}{2^3\cdot 3^3} = \frac{7}{2^3\cdot 3} = \frac{7}{24}$

Sol. Ex. 2.6.

- $\left(-\frac{3}{4}\right)^{-3}(2)^{-7}=-\left(\frac{4}{3}\right)^{3}\left(\frac{1}{2}\right)^{7}=-\frac{(2^2)^3}{3^3}\cdot\frac{1}{2^7}=-\frac{2^6}{3^3\cdot 2^7}=-\frac{1}{27\cdot 2}=-\frac{1}{54}$

  oppure:
  $$\left(-\frac{3}{4}\right)^{-3}(2)^{-7}=-\left(\frac{4}{3}\right)^{3}\left(\frac{1}{2}\right)^{7}=-\left(\frac{4}{3}\right)^{3}\left(\frac{1}{2}\right)^{3}\left(\frac{1}{2}\right)^{4}=-\left(\frac{4}{3}\cdot\frac{1}{2}\right)^{3}\left(\frac{1}{2}\right)^{4}=-\left(\frac{2}{3}\right)^{3}\left(\frac{1}{2}\right)^{4}=$$
  $$=-\frac{8}{27}\cdot\frac{1}{16}=-\frac{1}{54}.$$

- $\left(\frac{2}{7}\right)^{-4}\left(\frac{7}{4}\right)^{-2}=\left(\frac{7}{2}\right)^{4}\left(\frac{4}{7}\right)^{2}=\frac{7^4\cdot 4^2}{2^4\cdot 7^2}=\frac{7^2\cdot 2^4}{2^4}=7^2=49$

  oppure
  $$\left(\frac{2}{7}\right)^{-4}\left(\frac{7}{4}\right)^{-2}=\left(\frac{2}{7}\right)^{-2-2}\left(\frac{7}{4}\right)^{-2}=\left(\frac{2}{7}\cdot\frac{7}{4}\right)^{-2}\left(\frac{2}{7}\right)^{-2}=2^2\cdot\left(\frac{7}{2}\right)^{2}=7^2=49.$$

Sol. Ex. 2.7. **(B)**, infatti: $\left(127^{-3}\right)^4=(127)^{-3\cdot 4}=127^{-12}.$

Sol. Ex. 2.8.

- $2\cdot 2^{17}=2^{18}$
- $\left(2^{17}\right)^2=2^{34}$
- $\frac{1}{4}\cdot 2^{17}=2^{-2}\cdot 2^{17}=2^{15}$

Sol. Ex. 2.9. **(D)**, infatti**:**
$$\left(-\frac{7}{2}\right)^{-2}\cdot\left(\frac{5}{8}\right)^{3}\cdot 2^5\cdot 5^{-1}=\frac{7^{-2}}{2^{-2}}\cdot\frac{5^3}{\left(2^3\right)^3}\cdot 2^5\cdot 5^{-1}=2^{2-9+5}\cdot 5^{3-1}\cdot 7^{-2}=2^{-2}\cdot 5^2\cdot 7^{-2}=\left(\frac{5}{2\cdot 7}\right)^2=\left(\frac{5}{14}\right)^2.$$

Sol. Ex. 2.10. $a=3.75416\cdot 10^{-4}$, $b=4.673251\cdot 10$, $c=3.25241\cdot 10^{5}$

Sol. Ex. 2.11.

- $0.00002\cdot 35\cdot 7=(2\cdot 10^{-5})\cdot 7\cdot(3.5\cdot 10)=14\cdot 3.5\cdot 10^{-4}=49\cdot 10^{-4}=4.9\cdot 10^{-3}$
- $36782\cdot 0.00003=(3.6782\cdot 10^4)\cdot(3\cdot 10^{-5})=11.0346\cdot 10^{-1}=1.10346\cdot 10^0$
- $(0.08)^2:3.2=\left(8\cdot 10^{-2}\right)^2:3.2=\left(2^3\cdot 10^{-2}\right)^2\cdot\frac{10}{2^5}=2^6\cdot 10^{-4}\cdot\frac{10}{2^5}=2\cdot 10^{-3}$

Sol. Ex. 2.12.

- $\sqrt{\dfrac{2}{5}} \cdot \sqrt[3]{\dfrac{5}{2}} = 2^{\frac{1}{2}} \cdot 5^{-\frac{1}{2}} \cdot 5^{\frac{1}{3}} \cdot 2^{-\frac{1}{3}} = 2^{\frac{1}{2}-\frac{1}{3}} \cdot 5^{-\frac{1}{2}+\frac{1}{3}} = 2^{\frac{1}{6}} \cdot 5^{-\frac{1}{6}} = \sqrt[6]{\dfrac{2}{5}}.$
- $\sqrt[3]{\dfrac{2}{15}} \cdot \sqrt[4]{\dfrac{15}{2}} = 2^{\frac{1}{3}} \cdot 15^{-\frac{1}{3}} \cdot 15^{\frac{1}{4}} \cdot 2^{-\frac{1}{4}} = 2^{\frac{1}{3}-\frac{1}{4}} \cdot 15^{-\frac{1}{3}+\frac{1}{4}} = 2^{\frac{1}{12}} \cdot 15^{-\frac{1}{12}} = \left(\dfrac{2}{15}\right)^{\frac{1}{12}} = \sqrt[12]{\dfrac{2}{15}}.$
- $\sqrt{\dfrac{3}{4}} : \sqrt[4]{\dfrac{3}{2}} = 3^{\frac{1}{2}} \cdot 2^{-1} \cdot 2^{\frac{1}{4}} \cdot 3^{-\frac{1}{4}} = 2^{-1+\frac{1}{4}} \cdot 3^{\frac{1}{2}-\frac{1}{4}} = 2^{-\frac{3}{4}} \cdot 3^{\frac{1}{4}} = \sqrt[4]{\dfrac{3}{8}}$

Sol. Ex. 2.13.

- $\sqrt{200} \cdot \sqrt[3]{1\,000} : \sqrt[3]{8\,000\,000} = (2 \cdot 10^2)^{\frac{1}{2}} \, (10^3)^{\frac{1}{3}} \, (2^3 \cdot 10^6)^{-\frac{1}{3}} = 2^{\frac{1}{2}-1} \cdot 10^{1+1-2} = 2^{-\frac{1}{2}} = \dfrac{1}{\sqrt{2}}$
- $2^{1.2} \cdot 2^{-1} = 2^{\frac{12}{10}} \cdot 2^{-1} = 2^{\frac{6}{5}-1} = 2^{\frac{1}{5}} = \sqrt[5]{2}$, oppure $2^{1.2} \cdot 2^{-1} = 2^{1.2-1} = 2^{0.2} = 2^{\frac{1}{5}} = \sqrt[5]{2}$

Sol. Ex. 2.14.

- $\sqrt[5]{4} \cdot \sqrt[5]{8} = (4 \cdot 8)^{\frac{1}{5}} = (2^2 \cdot 2^3)^{\frac{1}{5}} = (2^5)^{\frac{1}{5}} = 2$
- $\sqrt[6]{\dfrac{a}{b}} \cdot \sqrt[6]{\dfrac{a^3}{b^3}} \cdot \sqrt[6]{\dfrac{b^2}{a^2}} = \left(\dfrac{a}{b} \cdot \dfrac{a^3}{b^3} \cdot \dfrac{b^2}{a^2}\right)^{\frac{1}{6}} = \left(\dfrac{a^4 b^2}{a^2 b^4}\right)^{\frac{1}{6}} = \left(\dfrac{a^2}{b^2}\right)^{\frac{1}{6}} = \left(\dfrac{a}{b}\right)^{\frac{2}{6}} = \sqrt[3]{\dfrac{a}{b}}$
- $\sqrt[6]{(a+2b)^4} \cdot \sqrt[6]{\dfrac{1}{a+2b}} \cdot \sqrt[6]{(a+2b)^5} = (a+2b)^{\frac{4}{6}-\frac{1}{6}+\frac{5}{6}} = (a+2b)^{\frac{4}{3}} = \sqrt[3]{(a+2b)^4} =$
$$= (a+2b)\,\sqrt[3]{(a+2b)}$$

Sol. Ex. 2.15. **(C)**, infatti: $\sqrt[4]{\dfrac{7^{-2}}{4^{-4}}} = \dfrac{7^{-\frac{2}{4}}}{4^{-\frac{4}{4}}} = 7^{-\frac{1}{2}} \cdot 4 = \dfrac{4}{\sqrt{7}}.$

Sol. Ex. 2.16. **(B)**, infatti:

$$\sqrt{2} : \sqrt[3]{12} = 2^{\frac{1}{2}} \cdot (2^2 \cdot 3)^{-\frac{1}{3}} = 2^{\frac{1}{2}-\frac{2}{3}} \cdot 3^{-\frac{1}{3}} = 2^{-\frac{1}{6}} \cdot 3^{-\frac{1}{3}} = 2^{-\frac{1}{6}} \cdot 3^{-\frac{2}{6}} = \frac{1}{(2 \cdot 3^2)^{\frac{1}{6}}} = \frac{1}{\sqrt[6]{18}}.$$

Sol. Ex. 2.17.

- $\sqrt[3]{-\dfrac{343}{8}} = \sqrt[3]{\left(-\dfrac{7}{2}\right)^3} = -\dfrac{7}{2}$
- $\sqrt[5]{-\dfrac{243}{160}} = \sqrt[5]{-\left(\dfrac{3}{2}\right)^5 \cdot \dfrac{1}{5}} = -\dfrac{3}{2\sqrt[5]{5}}$

- $\sqrt[2]{-\dfrac{4}{49}}$ non ha significato: la radice ha indice pari e il radicando è negativo.

Sol. Ex. 2.18. Gli indici sono rispettivamente 4, 2 e 3: il loro minimo comune multiplo è 12. Si riducono tutti i radicali all'indice 12:

$$\sqrt[4]{3} = \sqrt[12]{3^3} \quad ; \quad \sqrt{7} = \sqrt[12]{7^6} \quad ; \quad \sqrt[3]{2} = \sqrt[12]{2^4}$$

Sol. Ex. 2.19. Si riducono i radicali allo stesso indice: $\sqrt[5]{9} = \sqrt[15]{9^3}$ e $\sqrt[3]{2} = \sqrt[15]{2^5}$.
Da $9^3 > 2^5$ si ricava

$$\sqrt[5]{9} > \sqrt[3]{2}.$$

Sol. Ex. 2.20. Poiché $2 = \sqrt[3]{2^3} = \sqrt[3]{8}$ e $5 < 8$ si ha $\sqrt[3]{5} < 2$. Inoltre $\sqrt{30} > \sqrt{25} = 5 > 2$. Quindi

$$\sqrt[3]{5} < 2 < \sqrt{30}.$$

Lo stesso risultato si ottiene riducendo i radicali all'indice 6: $2 = \sqrt[6]{2^6}$, $\sqrt[3]{5} = \sqrt[6]{5^2}$, $\sqrt{30} = \sqrt[6]{(30)^3}$ e osservando che $5^2 = 25 < 2^6 = 64 < 30^3 = 27 \cdot 10^3$.

Sol. Ex. 2.21. Gli indici sono rispettivamente 3, 4, 6 e 2. Il minimo indice comune è 12.

$$\sqrt[3]{a^2} = \sqrt[12]{a^8} \quad , \quad \sqrt[4]{ab} = \sqrt[12]{a^3b^3} \quad , \quad \sqrt[6]{a} = \sqrt[12]{a^2} \quad , \quad \sqrt{b^3} = \sqrt[12]{b^{18}}.$$

Sol. Ex. 2.22. **(B)**, infatti: $\sqrt[3]{5\sqrt{5}} = \left(5 \cdot 5^{\frac{1}{2}}\right)^{\frac{1}{3}} = 5^{\frac{1}{3}(1+\frac{1}{2})} = 5^{\frac{1}{3}\cdot\frac{3}{2}} = 5^{\frac{1}{2}} = \sqrt{5}.$

Sol. Ex. 2.23.

- $\left(1 - \dfrac{1}{3}\right)\sqrt[4]{\dfrac{9}{16}} = \dfrac{2}{3}\sqrt{\dfrac{3}{4}} = \dfrac{2}{3 \cdot 2}\sqrt{3} = \dfrac{1}{\sqrt{3}}$
- $\dfrac{64^{-\frac{1}{2}} + \left(\dfrac{1}{4}\right)^{\frac{3}{2}} - 8^{-1}}{9^{-\frac{3}{2}} \cdot 27^{\frac{2}{3}}} = \dfrac{(2^6)^{-\frac{1}{2}} + (2^{-2})^{\frac{3}{2}} - (2^3)^{-1}}{(3^2)^{-\frac{3}{2}}(3^3)^{\frac{2}{3}}} = \dfrac{2^{-3} + 2^{-3} - 2^{-3}}{3^{-3} \cdot 3^2} = \dfrac{2^{-3}}{3^{-1}} = \dfrac{3}{8}.$

Sol. Ex. 2.24.

- Tenendo conto che $28 = 2^2 \cdot 7$, $63 = 3^2 \cdot 7$, $567 = 3^4 \cdot 7$, si ottiene:
  $\sqrt{28} + \sqrt{63} - 8\sqrt{7} + \sqrt{567} = 2\sqrt{7} + 3\sqrt{7} - 8\sqrt{7} + 9\sqrt{7} = 6\sqrt{7}$
- Tenendo conto che $12 = 2^2 \cdot 3$, si ottiene:
  $\left(2\sqrt{3} - \sqrt{12} + \sqrt{21}\right)\sqrt{3} = \left(2\sqrt{3} - 2\sqrt{3} + \sqrt{21}\right)\sqrt{3} = \sqrt{21}\sqrt{3} = \sqrt{3^2 \cdot 7} = 3\sqrt{7}.$
  Alternativamente, calcolando prima il prodotto:
  $\left(2\sqrt{3} - \sqrt{12} + \sqrt{21}\right)\sqrt{3} = 6 - \sqrt{36} + \sqrt{3^2 \cdot 7} = 6 - 6 + 3\sqrt{7} = 3\sqrt{7}.$

Sol. Ex. 2.25.

- $81 = 3^4 \qquad$ quindi $\quad \log_3 81 = 4.$
- $256 = 2^8 \quad$ quindi $\quad \log_2 256 = 8.$
- $-125 < 0 \quad$ quindi $\quad \log_5(-125)$ non ha significato.
- $243 = 3^5 \quad$ quindi $\quad \log_3 \dfrac{1}{243} = \log_3 (3^{-5}) = -5.$

Sol. Ex. 2.26.

- $c = 5^1 = 5$
- $c = (3)^{\frac{3}{4}} = \sqrt[4]{3^3} = \sqrt[4]{27}$
- $c = \left(\dfrac{2}{3}\right)^{\frac{5}{2}} = \sqrt{\left(\dfrac{2}{3}\right)^5} = \left(\dfrac{2}{3}\right)^2 \sqrt{\dfrac{2}{3}}$

Sol. Ex. 2.27.

- $c = \left(\dfrac{1}{27}\right)^{-\frac{1}{3}} = (27)^{\frac{1}{3}} = 3$
- $c = \left(\sqrt{\dfrac{1}{3}}\right)^2 = \dfrac{1}{3}$
- $c = \left(\dfrac{1}{8}\right)^{\frac{2}{3}} = \left(\dfrac{1}{2^3}\right)^{\frac{2}{3}} = \dfrac{1}{2^2} = \dfrac{1}{4}$

Sol. Ex. 2.28.

- $a^3 = 8 = 2^3$ significa $a = 2$
- $b = 1/2$
- $a^{-3} = 8 = 2^3$ significa $a^{-1} = 2$ cioè $a = 2^{-1} = \dfrac{1}{2}$
- $c = \left(\dfrac{1}{2}\right)^{-\frac{2}{3}} = 2^{\frac{2}{3}} = \sqrt[3]{4}$
- $a^{\frac{2}{3}} = 4$ significa $a = 4^{\frac{3}{2}} = (2^2)^{\frac{3}{2}} = 8$

Sol. Ex. 2.29. $2 + \log_5 \dfrac{1}{15} = \log_5 5^2 + \log_5 \dfrac{1}{15} = \log_5 \dfrac{25}{15} = \log_5 \dfrac{5}{3}$

Sol. Ex. 2.30.
$2\log_2 a + \dfrac{1}{2}\log_2 b + 3\log_2 c = \log_2 a^2 + \log_2 b^{\frac{1}{2}} + \log_2 c^3 = \log_2\left(a^2 b^{\frac{1}{2}} c^3\right) = \log_2\left(a^2 c^3 \sqrt{b}\right)$

Sol. Ex. 2.31.

- $10^{\log_{10} 2} = 2.$
- $2^{1/\log_{10} 2} = 2^{\log_2 10} = 10$
- $2^{\log_{10} 10} = 2^1 = 2.$

Sol. Ex. 2.32.

- $3^{-\log_3 7} = 3^{\log_3 \left(7^{-1}\right)} = 1/7.$
- $4^{\log_{1/4} 2} = 4^{-\log_4 2} = \left(4^{\log_4 2}\right)^{-1} = 1/2$ oppure $4^{\log_{1/4} 2} = (1/4)^{-\log_{1/4} 2} = (1/4)^{\log_{1/4}(1/2)} = 1/2.$
- $5^{2\log_5 3 + 3\log_5 2} = 5^{\log_5 3^2 + \log_5 2^3} = 5^{\log_5\left(3^2 \cdot 2^3\right)} = 5^{\log_5 72} = 72.$

Sol. Ex. 2.33. $\log_3 \dfrac{27\sqrt[3]{3}}{\sqrt{\sqrt[4]{9}}} = \log_3 \dfrac{3^{3+\frac{1}{3}}}{(3^2)^{\frac{1}{8}}} = \log_3 (3)^{\frac{10}{3}-\frac{1}{4}} = \log_3 (3)^{\frac{37}{12}} = \dfrac{37}{12}.$

Sol. Ex. 2.34. (B), infatti
$\left(2\cdot\sqrt[3]{\dfrac{2}{\sqrt{2}}}\right) = 2\cdot\left(2^{\frac{1}{2}}\right)^{\frac{1}{3}} = 2^{1+\frac{1}{6}} = 2^{\frac{7}{6}} = \left(\dfrac{1}{2}\right)^{-\frac{7}{6}}$ e quindi $\log_{\frac{1}{2}}\left(2\cdot\sqrt[3]{\dfrac{2}{\sqrt{2}}}\right) = -\dfrac{7}{6}.$

Sol. Ex. 2.35.
Calcolo diretto del valore: $\log_6 9 = d$ significa $6^d = 9$ cioè $(6^2)^{\frac{d}{2}} = 9$ e quindi $\log_{36} 9 = \dfrac{d}{2}.$
Oppure, usando la formula del cambiamento di base: $\log_{36} 9 = \dfrac{\log_6 9}{\log_6 36} = \dfrac{\log_6 9}{2} = \dfrac{d}{2}.$

Sol. Ex. 2.36. (A). Usare la formula del cambiamento di base riscritta come $\log_a x = \log_a b \cdot \log_b x$:

$$\log_2 80 = \log_2 16 \cdot \log_{16} 80 = 4c.$$

Sol. Ex. 2.37.

- $\log_2 6 + \log_{\frac{1}{2}} 9 = \log_2 6 - \log_2 9 = \log_2 \left(\frac{6}{9}\right) = \log_2 \left(\frac{2}{3}\right) = 1 - \log_2 3$
- $\log_5 200 - 3\log_5 2 = \log_5 200 - \log_5 2^3 = \log_5 \left(\frac{200}{8}\right) = \log_5 25 = 2$
- $\left(\sqrt{3}\right)^c = 3^{c/2}$: quindi per ogni $b > 0$: $\log_{\sqrt{3}} b = 2\log_3 b$. Si ha allora $\log_{\frac{1}{3}} 4 + \log_{\sqrt{3}} 144 =$
  $= -\log_3 4 + 2\log_3 144 = -\log_3 4 + 4\log_3 12 = -\log_3 4 + 4\left(\log_3 4 + 1\right) = 3\log_3 4 + 4$

Sol. Ex. 2.38. Ricordando le proprietà dei logaritmi:

- $2 > 1 \quad , \quad 5 < 7$ e quindi $\log_2 5 < \log_2 7$.
- $\frac{1}{2} < 1 \quad , \quad 3 < 207$ e quindi $\log_{\frac{1}{2}} 207 < \log_{\frac{1}{2}} 3$.
- $\log_{\frac{1}{3}} 241 = -\log_3 241 < 0 \quad , \quad \log_3 23 > 0$ e quindi $\log_{\frac{1}{3}} 241 < \log_3 23$

Sol. Ex. 2.39. I logaritmi hanno basi diverse, ma entrambi hanno per base e per argomento una potenza di 2. Per calcolo diretto si ha: $256 = 2^8 = 4^4 = (2^3)^{\frac{8}{3}}$ e quindi $\log_4 \frac{1}{256} = -4$, mentre $\log_8 \frac{1}{256} = -\frac{8}{3}$ cioè $\log_4 \frac{1}{256} < \log_8 \frac{1}{256}$.
In realtà non è necessario fare i conti, basta osservare che

$$\begin{aligned}
&\log_4 256 = \log_4 8 \cdot \log_8 256 \\
&\log_4 8 > \log_4 4 = 1 \text{ (poiché la base 4 del logaritmo è } > 1 \text{ e } 8 > 4) \\
&\log_8 256 > \log_8 1 = 0
\end{aligned}$$

per concludere che $\log_4 256 > \log_8 256$ e quindi $-\log_4 256 < -\log_8 256$.

Sol. Ex. 2.40. Tutti i logaritmi sono in base $2 > 1$ e quindi da $\frac{1}{6} < 3 < 8$ si ricava $\log_2 \frac{1}{6} < \log_2 3 < \log_2 8$; tutte le potenze sono in base $4 > 1$ e quindi $4^{\log_2 \frac{1}{6}} < 4^{\log_2 3} < 4^{\log_2 8}$.
Oppure, calcolando direttamente i valori:

$$\begin{aligned}
&4^{\log_2 3} = 2^{2\log_2 3} = 2^{\log_2 9} = 9 \\
&4^{\log_2 8} = 4^3 = 64 \\
&4^{\log_2 \frac{1}{6}} = 2^{2\log_2 \frac{1}{6}} = 2^{\log_2 \frac{1}{36}} = \frac{1}{36}
\end{aligned} \quad : \quad \frac{1}{36} < 9 < 64 \quad \text{cioè} \quad 4^{\log_2 \frac{1}{6}} < 4^{\log_2 3} < 4^{\log_2 8}.$$

Sol. Ex. 2.41. Tutte le potenze sono in base $\frac{1}{3} < 1$ e quindi le disuguaglianze valide per gli esponenti si rovesciano per le potenze. Gli esponenti sono:

$$4\log_{\frac{1}{3}} 2 = -4\log_3 2 = -\log_3 2^4, \qquad 4\log_3 2 = \log_3 2^4, \qquad \log_{\frac{1}{3}} \frac{3}{7} = -\log_3 \frac{3}{7}$$

e si ha

$$-\log_3 2^4 < -\log_3 \frac{3}{7} < \log_3 2^4;$$

quindi

$$\left(\frac{1}{3}\right)^{4\log_3 2} < \left(\frac{1}{3}\right)^{\log_{\frac{1}{3}} \frac{3}{7}} < \left(\frac{1}{3}\right)^{4\log_{\frac{1}{3}} 2}.$$

Lo stesso risultato si ottiene calcolando direttamente i valori delle potenze

$$\begin{aligned}
&\left(\frac{1}{3}\right)^{4\log_{\frac{1}{3}} 2} = \left(\frac{1}{3}\right)^{\log_{\frac{1}{3}}\left(2^4\right)} = 2^4 = 16 \\
&\left(\frac{1}{3}\right)^{4\log_3 2} = \left(\frac{1}{3}\right)^{\log_3\left(2^4\right)} = \left(\frac{1}{3}\right)^{\log_{\frac{1}{3}}\left(2^{-4}\right)} = 2^{-4} = \frac{1}{16} \\
&\left(\frac{1}{3}\right)^{\log_{\frac{1}{3}}\left(\frac{3}{7}\right)} = \frac{3}{7}
\end{aligned} \quad .$$

Le disuguaglianze $\frac{1}{16} < \frac{3}{7} < 16$ si rileggono $\left(\frac{1}{3}\right)^{4\log_3 2} < \left(\frac{1}{3}\right)^{\log_{\frac{1}{3}} \frac{3}{7}} < \left(\frac{1}{3}\right)^{4\log_{\frac{1}{3}} 2}$.