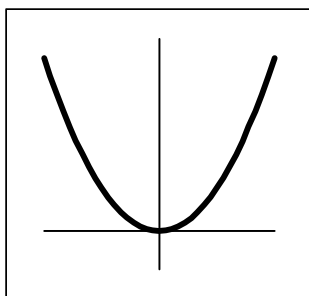


Argomento 3

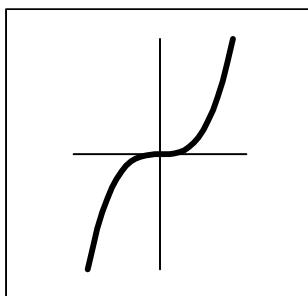
Suggerimenti

Suggerimento generale:

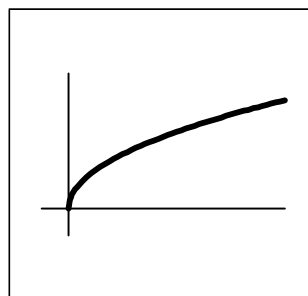
utilizzare i seguenti grafici per ricordare il comportamento delle funzioni elementari



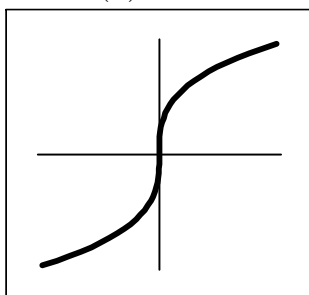
$$f(x) = x^{2n}$$



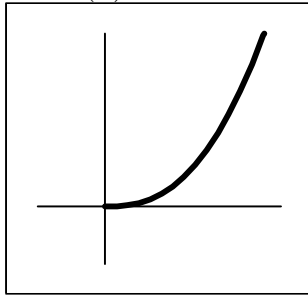
$$f(x) = x^{2n+1}$$



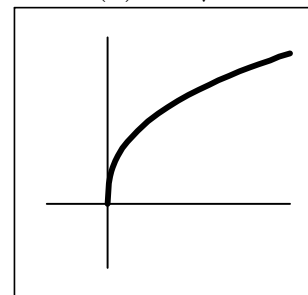
$$f(x) = \sqrt[n]{x}$$



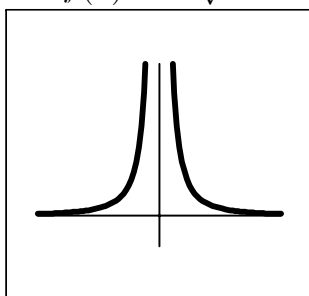
$$f(x) = \frac{1}{\sqrt[n]{x}}$$



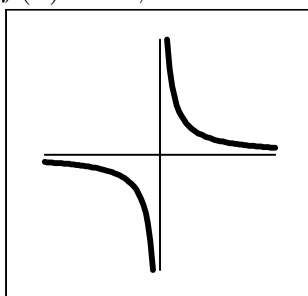
$$f(x) = x^\alpha, \alpha \text{ reale } > 1$$



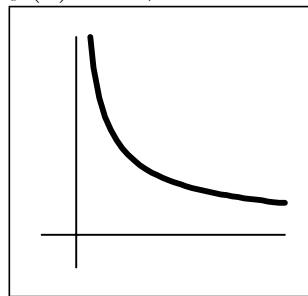
$$f(x) = x^\alpha, 0 < \alpha < 1$$



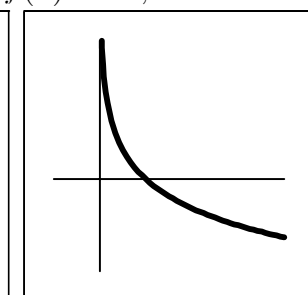
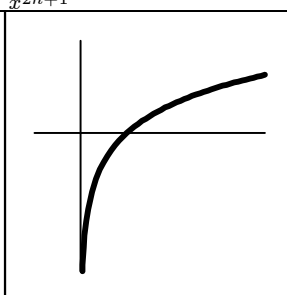
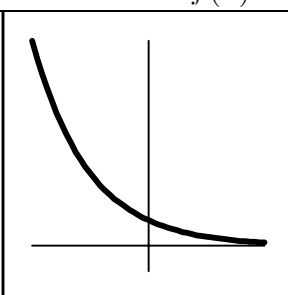
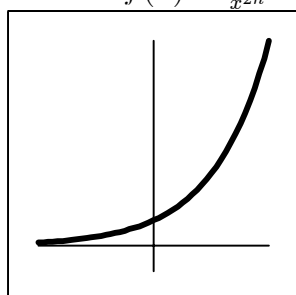
$$f(x) = \frac{1}{x^{2n}} = x^{-2n}$$



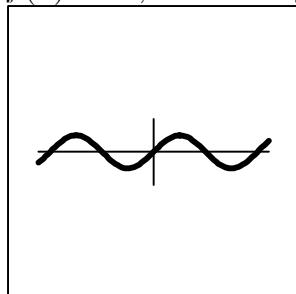
$$f(x) = \frac{1}{x^{2n+1}} = x^{-2n-1}$$



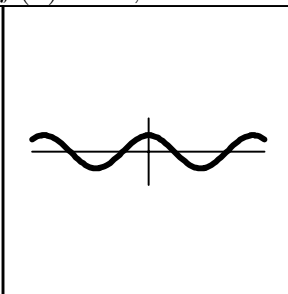
$$f(x) = x^\alpha, \alpha \text{ reale } < 0$$



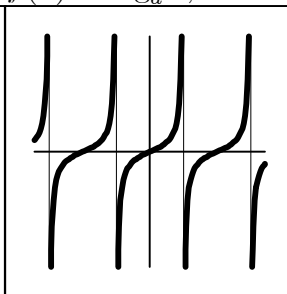
$$f(x) = a^x, \text{ con } a > 1 \quad f(x) = a^x, 0 < a < 1 \quad f(x) = \log_a x, a > 1 \quad f(x) = \log_a x, 0 < a < 1$$



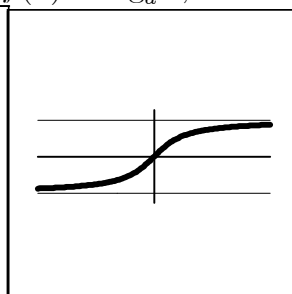
$$f(x) = \sin x$$



$$f(x) = \cos x$$



$$f(x) = \tan x$$



$$f(x) = \arctan x$$

Suggerimenti Ex. 3.1

17. Si noti che $\lim_{x \rightarrow 1^+} x - 1 = 0^+$ quindi $\lim_{x \rightarrow 1^+} \frac{1}{x-1} = +\infty$.
19. Si noti che $\lim_{x \rightarrow +\infty} \log_2 x = +\infty$ e $\lim_{x \rightarrow +\infty} \log_{\frac{1}{2}} x = -\infty$ (oppure che $\log_{\frac{1}{2}} x = -\log_2 x$)
21. Si noti che $\lim_{x \rightarrow -\infty} 3x + 2 = -\infty$ e $\lim_{x \rightarrow -\infty} e^{2x} + 4 = 4$.
24. Si noti che $\lim_{x \rightarrow +\infty} \log(e + e^{-x}) = 1$ e $\lim_{x \rightarrow +\infty} 1 - \frac{1}{x} = 1$.
26. Si noti che $\lim_{x \rightarrow +\infty} \left(\arctan x + \frac{1}{x} \right) = \frac{\pi}{2} + 0$.
27. Si noti che $\lim_{x \rightarrow +\infty} 3^{\frac{1}{x}} = 3^0 = 1$ e $\lim_{x \rightarrow +\infty} \sqrt{x^2 - 2} = +\infty$.

Suggerimenti Ex. 3.2

1. Si noti che $\lim_{x \rightarrow 0^+} \frac{x^2 + 2x}{2x^5 - 5x^2} = \lim_{x \rightarrow 0^+} \frac{x + 2}{x(2x^3 - 5)} = \dots$
3. Si noti che $\lim_{x \rightarrow 0} (\cos x)^2 = 1$ e $\lim_{x \rightarrow 0} \left(\frac{1}{x^3} \right)^2 = \lim_{x \rightarrow 0} \frac{1}{x^6} = +\infty$.
4. Si noti che $\lim_{x \rightarrow 2^+} x - 2 = 0^+$ e $\lim_{x \rightarrow 2^-} x - 2 = 0^-$
6. Si noti che $\lim_{x \rightarrow 0^-} \sqrt[3]{x^3 + x} = 0^-$

Suggerimenti Ex. 3.3

1. Si ricordi che $a^b = e^{b \log a}$ ($a > 0$). Si noti che $\lim_{x \rightarrow +\infty} (x^3 - 1) \log(x^2 + 1) = +\infty$
2. Si noti che $\lim_{x \rightarrow 1} \log \left(\frac{x^2 + 2x}{2x^3 - 1} \right) = \log 3$ e $\lim_{x \rightarrow 1} 2x^2 = 2$
3. Si noti che $\lim_{x \rightarrow +\infty} \log(\log x) = +\infty$ e $\lim_{x \rightarrow +\infty} \arctan(x) = \frac{\pi}{2}$

Suggerimenti Ex. 3.4

1. Si noti che valgono le disuguaglianze $\frac{1}{2x+1} \leq \frac{1}{2x - \sin x} \leq \frac{1}{2x-1}$ in un intorno $U(+\infty)$ di $+\infty$ in cui tutti i denominatori sono definiti e positivi, ad esempio in $\left(\frac{1}{2}, +\infty \right)$.

Argomento 3

Soluzioni

Soluzioni Ex. 3.1

1.	4	2.	1	3.	$+\infty$
4.	1	5.	$+\infty$	6.	$\frac{2}{3}$
7.	0	8.	0	9.	$-\infty$
10.	$-\infty$	11.	$+\infty$	12.	3
13.	$-\infty$	14.	0	15.	5
16.	0	17.	$+\infty$	18.	0
19.	$-\infty$	20.	-2	21.	$-\infty$
22.	$-\infty$	23.	0	24.	1
25.	0	26.	1	27.	0

Soluzioni Ex. 3.2

1.	non esiste	2.	$+\infty$	3.	$+\infty$
4.	non esiste	5.	$-\infty$	6.	$-\infty$

Soluzioni Ex. 3.3

1.	$+\infty$	2.	9	3.	$+\infty$
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Soluzioni Ex. 3.4

1.	0	2.	0	3.	$+\infty$
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Soluzioni Ex. 3.5

- | | | |
|---------------------------------|----------------------|---------------------------|
| 1. 4 | 2. 11 | 3. $\frac{4}{\pi}$ |
| 4. 1 | 5. non esiste | 6. $-\infty$ |
| 7. $+\infty$ | 8. $-\infty$ | 9. 0 |
| 10. 0 | 11. -1 | 12. $+\infty$ |
| 13. $\frac{\sqrt{2}}{2}$ | 14. 1 | 15. 0 |