Argomento 9
Suggerimenti

Ex. 9.4

1) Integrare per parti scegliendo $\log x$ come fattore finito.

2) Se D indica la derivata, osservare che $D\left(e^x - 1\right) = e^x$.

3) Integrare per parti scegliendo e^x come fattore differenziale.

4) Osservare che $D\left(\log\left(x+1\right)\right) = \dfrac{1}{x+1}$.

5) Osservare che $D\left(x - \cos x\right) = 1 + \sin x$

6) Integrare per parti scegliendo x come fattore finito.

7) Osservare che $\dfrac{x+1}{x^2+1} = \dfrac{1}{2}\dfrac{2x}{x^2+1} + \dfrac{1}{x^2+1}$.

8) Osservare che $D\left(\sqrt{x}\right) = \dfrac{1}{2\sqrt{x}}$.

9) Osservare che $\dfrac{2}{x^2-2x} = \dfrac{1}{x-2} - \dfrac{1}{x}$.

10) Per sostituzione ponendo $t = e^x$.

11) Per sostituzione ponendo $t = \sqrt{x+1}$.

12) Osservare che $D\left(\log\left(x+1\right)\right) = \dfrac{1}{x+1}$.

Ex. 9.5

3) I grafici di $y = \sqrt{x}$ e di $y = \dfrac{2}{x}$ si intersecano nel punto di ascissa $\sqrt[3]{4}$.

4) I grafici di $y = |x|$ e di $y = 12 - x^2$ si intersecano nei punti di ascissa -3 e 3.
Quindi $A = \displaystyle\int_{-3}^{3} \left(12 - x^2 - |x|\right)\, dx$. Poichè la regione è simmetrica rispetto all'asse y si ha che $A = 2\displaystyle\int_0^3 \left(12 - x^2 - x\right)\, dx$.

8) La funzione $f\left(x\right) = -x\left(x+2\right)$ cambia segno in $x = 0$.

9) La regione è simmetrica rispetto ad $x = \dfrac{3}{2}$, e la funzione $f\left(x\right) = x^3 - 3x$ cambia segno in $x = 0$ e in $x = 3$.

11) I grafici delle curve $y = 2\left(1 - x^2\right)$ e $y = \left(x-1\right)^2$ si intersecano in $x = -\dfrac{1}{3}$ e $x = 1$.

14) $\min(x,x^2) = \begin{cases} x^2 & \text{se } 0 \leq x \leq 1 \\ x & \text{se } 1 \leq x \leq 2 \end{cases}$, e quindi $\max(x,x^2) = \begin{cases} x & \text{se } 0 \leq x \leq 1 \\ x^2 & \text{se } 1 \leq x \leq 2 \end{cases}$, per cui $|\max(x,x^2) - \min(x,x^2)| = \begin{cases} x - x^2 & \text{se } 0 \leq x \leq 1 \\ x^2 - x & \text{se } 1 \leq x \leq 2 \end{cases}$.

15) $(2y + x - 4)(2y - 3\sqrt{x}) = 4\left(y - \frac{4-x}{2}\right)\left(y - \frac{3}{2}\sqrt{x}\right)$ ed il prodotto è negativo quando i segni dei due fattori sono discordi, cioè quando il punto (x,y) sta contemporaneamente sopra alla retta $y = \frac{4-x}{2}$ e sotto alla curva $y = \frac{3}{2}\sqrt{x}$, o viceversa. Le curve si incontrano in $x = 1$. Perciò la regione si può anche descrivere come

$$\left\{(x,y) : 0 \leq x \leq 1,\ \frac{3}{2}\sqrt{x} \leq y \leq \frac{4-x}{2}\right\} \cup \left\{(x,y) : 1 \leq x \leq 4,\ \frac{4-x}{2} \leq y \leq \frac{3}{2}\sqrt{x}\right\}.$$

Ex. 9.6

La retta tangente al grafico della funzione $F(x)$ nel punto di ascissa x_0 ha equazione

$$y = F(x_0) + F'(x_0)(x - x_0).$$

Quindi occorre calcolare $F(1)$ e $F'(1)$. Per calcolare la derivata può essere utile ricordare il Teorema fondamentale del calcolo integrale.

Ex. 9.7

Occorre calcolare: $\frac{1}{\mathrm{e}-1}\int_1^{\mathrm{e}} 4x \log x \, dx$. Per determinare una primitiva di $4x \log x$ integrare per parti scegliendo $\log x$ come fattore finito.

Soluzioni Esercizi

Sol. Ex. 9.1

1) $\displaystyle\int \frac{1}{\sqrt{x}}\,dx = 2\sqrt{x}+c \quad\Longrightarrow\quad \int_1^4 \frac{1}{\sqrt{x}}\,dx = 2$

2) $\displaystyle\int 3\sqrt{x}\,dx = 2\left(\sqrt{x}\right)^3+c \quad\Longrightarrow\quad \int_1^9 3\sqrt{x}\,dx = 52$

3) $\displaystyle\int \left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)dx = \frac{2}{3}\left(\sqrt{x}\right)^3-2\sqrt{x}+c \quad\Longrightarrow\quad \int_1^9 \left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)dx = \frac{40}{3}$

4) $\displaystyle\int \frac{x^2+1}{x^2}\,dx = x-\frac{1}{x}+c \quad\Longrightarrow\quad \int_{-2}^{-1} \frac{x^2+1}{x^2}\,dx = \frac{3}{2}$

5) $\displaystyle\int \left(7x+\frac{1}{x}\right)dx = \frac{7}{2}x^2+\log|x|+c \quad\Longrightarrow\quad \int_{-2}^{-1} \left(7x+\frac{1}{x}\right)dx = -\frac{21}{2}-\log 2$

6) $\displaystyle\int \frac{3+x}{\sqrt{x}}\,dx = 6\sqrt{x}+\frac{2}{3}\left(\sqrt{x}\right)^3+c \quad\Longrightarrow\quad \int_3^4 \frac{3+x}{\sqrt{x}}\,dx = \frac{52}{3}-8\sqrt{3}$

7) $\displaystyle\int \frac{2x^4+2\sqrt{x}-3}{x}\,dx = \frac{x^4}{2}+4\sqrt{x}-3\log x+c \quad\Longrightarrow\quad \int_1^2 \frac{2x^4+2\sqrt{x}-3}{x}\,dx = \frac{7}{2}+4\sqrt{2}-3\log 2$

8) $\displaystyle\int (e^x+2)\,dx = e^x+2x+c \quad\Longrightarrow\quad \int_0^2 (e^x+2)\,dx = e^2+3$

9) $\displaystyle\int (1+\cos x)\,dx = x+\sin x+c \quad\Longrightarrow\quad \int_0^\pi (1+\cos x)\,dx = \pi$

10) $\displaystyle\int \frac{x-\sqrt{x}-2}{1+\sqrt{x}}\,dx = -2x+\frac{2}{3}\left(\sqrt{x}\right)^3+c \quad\Longrightarrow\quad \int_0^1 \frac{x-\sqrt{x}-2}{1+\sqrt{x}}\,dx = -\frac{4}{3}$

11) $\displaystyle\int e^x\,dx = e^x+c \quad\Longrightarrow\quad \int_{-1}^3 e^x\,dx = e^3-\frac{1}{e}$

12) $\displaystyle\int \frac{x^2+2}{x^2+1}\,dx = x+\arctan x+c \quad\Longrightarrow\quad \int_0^1 \frac{x^2+2}{x^2+1}\,dx = 1+\frac{\pi}{4}$

13) $\displaystyle\int \frac{7x+2}{x}\,dx = 7x+2\log|x|+c \quad\Longrightarrow\quad \int_1^e \frac{7x+2}{x}\,dx = 7e-5$

14) $\displaystyle\int \left(\cos x+\frac{3}{\sqrt{x}}\right)dx = \sin x+6\sqrt{x}+c \quad\Longrightarrow\quad \int_0^\pi \left(\cos x+\frac{3}{\sqrt{x}}\right)dx = 6\sqrt{\pi}$

15) $\displaystyle\int \frac{x^4-16}{x+2}\,dx = \frac{x^4}{4}-\frac{2}{3}x^3+2x^2-8x+c \quad\Longrightarrow\quad \int_{-1}^1 \frac{x^4-16}{x+2}\,dx = -\frac{52}{3}$

Sol. Ex. 9.2

1) $\displaystyle\int \sqrt{2-x}\,dx = -\frac{2}{3}\left(\sqrt{2-x}\right)^3 + c \quad\Longrightarrow\quad \int_0^2 \sqrt{2-x}\,dx = \frac{4}{3}\sqrt{2}$

2) $\displaystyle\int \frac{dx}{x-4} = \log|x-4| + c \quad\Longrightarrow\quad \int_0^3 \frac{dx}{x-4} = -2\log 2$

3) $\displaystyle\int \sin\left(\frac{x}{3}\right)\,dx = -3\cos\left(\frac{x}{3}\right) + c \quad\Longrightarrow\quad \int_\pi^{3\pi} \sin\left(\frac{x}{3}\right)\,dx = \frac{9}{2}$

4) $\displaystyle\int \frac{1}{(2x+1)^2}\,dx = -\frac{1}{2(2x+1)} + c \quad\Longrightarrow\quad \int_0^1 \frac{1}{(2x+1)^2}\,dx = \frac{1}{3}$

5) $\displaystyle\int e^{3-2x}\,dx = -\frac{1}{2}e^{3-2x} + c \quad\Longrightarrow\quad \int_0^1 e^{3-2x}\,dx = \frac{e^3-e}{2}$

6) $\displaystyle\int (3x-4)^5\,dx = \frac{1}{18}(3x-4)^6 + c \quad\Longrightarrow\quad \int_1^2 (3x-4)^5\,dx = \frac{7}{2}$

7) $\displaystyle\int \sin\left[2\pi(x-1)\right]\,dx = -\frac{\cos(2\pi x)}{2\pi} + c \quad\Longrightarrow\quad \int_0^{1/2} \sin\left[2\pi(x-1)\right]\,dx = \frac{1}{\pi}$

8) $\displaystyle\int \left(e^{x-1} + \sqrt[3]{x+1}\right)\,dx = e^{x-1} + \frac{3}{4}\left(\sqrt[3]{x+1}\right)^4 + c \quad\Longrightarrow\quad \int_0^1 \left(e^{x-1} + \sqrt[3]{x+1}\right)\,dx = \frac{1}{4} + \frac{3}{2}\sqrt[3]{2} - \frac{1}{e}$

9) $\displaystyle\int x^5\left(\frac{3}{x}+2\right)^5\,dx = \frac{1}{12}(2x+3)^6 + c \quad\Longrightarrow\quad \int_{-1}^{-1/2} x^5\left(\frac{3}{x}+2\right)^5\,dx = \frac{21}{4}$

Sol. Ex. 9.3

1) $\displaystyle\int \frac{x}{x^2+1}\,dx = \frac{1}{2}\log\left(x^2+1\right) + c \quad\Longrightarrow\quad \int_0^1 \frac{x}{x^2+1}\,dx = \frac{1}{2}\log 2$

2) $\displaystyle\int x^2\sin x^3\,dx = -\frac{1}{3}\cos x^3 + c \quad\Longrightarrow\quad \int_{-\pi}^{\pi} x^2\sin x^3\,dx = 0$

3) $\displaystyle\int \frac{-2x}{9-x^2}\,dx = \log\left|x^2-9\right| + c \quad\Longrightarrow\quad \int_{-5}^{-4} \frac{-2x}{9-x^2}\,dx = \log 7 - 4\log 2$

4) $\displaystyle\int x\cos(x^2)\,dx = \frac{1}{2}\sin x^2 + c \quad\Longrightarrow\quad \int_{-\pi/2}^{\pi/2} x\cos(x^2)\,dx = 0$

5) $\displaystyle\int \frac{x+1}{\sqrt{x^2+2x}}\,dx = \sqrt{x^2+2x} + c \quad\Longrightarrow\quad \int_1^3 \frac{x+1}{\sqrt{x^2+2x}}\,dx = \sqrt{3}\left(\sqrt{5}-1\right)$

6) $\displaystyle\int (2x+3)\, e^{x^2+3x}\, dx = e^{x^2+3x} + c \quad \Longrightarrow \quad \int_{-3}^{0} (2x+3)\, e^{x^2+3x}\, dx = 0$

7) $\displaystyle\int \sin x \sqrt[3]{\cos x}\, dx = -\frac{3}{4}\left(\sqrt[3]{\cos x}\right)^4 + c \quad \Longrightarrow \quad \int_{0}^{\pi/2} \sin x \sqrt[3]{\cos x}\, dx = \frac{3}{4}$

8) $\displaystyle\int \frac{2e^x}{1+e^x} \log\left(1+e^x\right)\, dx = \log^2\left(1+e^x\right) + c \quad \Longrightarrow \quad \int_{0}^{\log 2} \frac{2e^x}{1+e^x} \log\left(1+e^x\right)\, dx = \log^2 3 - \log^2 2$

9) $\displaystyle\int \frac{2x}{x+3}\, dx = 2x - 6\log|x+3| + c \quad \Longrightarrow \quad \int_{-1}^{1} \frac{2x}{x+3}\, dx = 4 - 6\log 2$

Sol. Ex. 9.4

1) $\displaystyle\int \frac{\log x}{\sqrt{x}}\, dx = 2\sqrt{x}\log x - 4\sqrt{x} + c \quad \Longrightarrow \quad \int_{1}^{e} \frac{\log x}{\sqrt{x}}\, dx = 4 - 2\sqrt{e}$

2) $\displaystyle\int \frac{e^x}{\sqrt{e^x-1}}\, dx = 2\sqrt{e^x-1} + c \quad \Longrightarrow \quad \int_{0}^{\log 4} \frac{e^x}{\sqrt{e^x-1}}\, dx = 2\sqrt{3}$

3) $\displaystyle\int e^x\,(x+2)\, dx = e^x(x+1) + c \quad \Longrightarrow \quad \int_{0}^{2} e^x\,(x+2)\, dx = 3e^2 - 1$

4) $\displaystyle\int \frac{\sin(\log(x+1))}{x+1}\, dx = -\cos\left(\log\left(x+1\right)\right) + c \quad \Longrightarrow \quad \int_{0}^{1} \frac{\sin(\log(x+1))}{x+1}\, dx = 1 - \cos\left(\log 2\right)$

5) $\displaystyle\int \frac{1+\sin x}{(x-\cos x)^3}\, dx = -\frac{1}{2\left(x-\cos x\right)^2} + c \quad \Longrightarrow \quad \int_{0}^{\pi} \frac{1+\sin x}{(x-\cos x)^3}\, dx = \frac{1}{2}\left(1 - \frac{1}{(\pi+1)^2}\right)$

6) $\displaystyle\int x\cos x\, dx = \cos x + x\sin x + c \quad \Longrightarrow \quad \int_{0}^{\frac{\pi}{2}} x\cos x\, dx = \frac{1}{2}\pi - 1$

7) $\displaystyle\int \frac{x+1}{x^2+1}\, dx = \frac{1}{2}\log\left(x^2+1\right) + \arctan x + c \quad \Longrightarrow \quad \int_{0}^{1} \frac{x+1}{x^2+1}\, dx = \frac{1}{2}\log 2 + \frac{1}{4}\pi$

8) $\displaystyle\int \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx = -2\cos\sqrt{x} + c \quad \Longrightarrow \quad \int_{0}^{\pi^2} \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx = 4$

9) $\displaystyle\int \frac{2}{x^2-2x}\, dx = \log|x-2| - \log|x| + c \quad \Longrightarrow \quad \int_{3}^{4} \frac{2}{x^2-2x}\, dx = \log 3 - \log 2$

10) $\displaystyle\int \frac{e^{2x}+2e^x}{e^x+1}\, dx = e^x + \log\left(e^x+1\right) + c \qquad \int_{0}^{1} \frac{e^{2x}+2e^x}{e^x+1}\, dx = e - 1 + \log\left(\frac{e+1}{2}\right)$

11) $\int x\sqrt{x+1}\ dx = -\frac{2}{3}\left(\sqrt{x+1}\right)^3 + \frac{2}{5}\left(\sqrt{x+1}\right)^5 + c \qquad \int_0^1 x\sqrt{x+1}\ dx = \frac{4}{15}\left(\sqrt{2}+1\right)$

12) $\int \frac{\log(x+1)}{x+1}\ dx = \frac{1}{2}\log^2(x+1) + c \qquad \Longrightarrow \qquad \int_0^3 \frac{\log(x+1)}{x+1}\ dx = 2\left(\log 2\right)^2$

Sol. Ex. 9.5

1) $A = \int_e^{e^2} \frac{1}{x} dx = 1$

2) $A = -\int_0^4 (x^2 - 4x) dx = \frac{32}{3}$

3) $A = \int_{\sqrt[3]{4}}^3 \left(\sqrt{x} - \frac{2}{x}\right) dx = 2\sqrt{3} - 2\ \log 3 - \frac{4}{3} + \frac{4}{3}\ \log 2$

4) $A = 2\int_0^3 (-x^2 + 12 - x) dx = 45$

5) $A = \int_{-9}^0 e^{-x} dx = e^9 - 1$

6) $A = \int_0^3 \left[(6-2x) - \left(\frac{1}{\sqrt{x+1}} - 1\right)\right] dx = 10$

7) $A = \int_0^1 \left(2 + \frac{1}{x+1} + \sqrt[3]{x}\right) dx = \frac{11}{4} + \log 2$

8) $A = \int_{-2}^1 \left|-x(x+2)\right|\ dx = \frac{8}{3}$

9) $A = \int_{-1}^4 \left|x^2 - 3x\right|\ dx = \frac{49}{6}$

10) $A = \int_0^1 \left(\frac{1}{2x+1} + x\right) dx = \frac{1+\log 3}{2}$

11) $A = \int_{-1}^1 \left|3x^2 - 2x - 1\right|\ dx = \frac{64}{27}$

12) $A = \int_{-\pi/2}^{\pi/4} \left[\cos x - \frac{2\sqrt{2}}{3\pi}\left(x + \frac{\pi}{2}\right)\right] dx = 1 + \frac{\sqrt{2}}{2} - \frac{3\sqrt{2}}{16}\pi$

13) $A = \left(\sqrt{3} - 1\right) \int_1^4 \sqrt{x-1}\, dx = 6 - 2\sqrt{3}$

14) $A = \int_0^2 \left|x^2 - x\right| \, dx = 1$

15) $A = \int_0^4 \left|2 - \frac{x}{2} - \frac{3}{2}\sqrt{x}\right| \, dx = \frac{11}{2}$

Sol. Ex. 9.6 C.

Sol. Ex. 9.7 D.