

In [1]:

```
import pandas as pd
import matplotlib.pyplot as plt
import statsmodels.api as sm
import numpy as np
import scipy.stats as st
```

Gennaio 2019

Esercizio 0

$X \in (-1, 1)$ con $P(X = 1) = p$

0.1

$$1 - P(X = 1) = P(X = -1) = 1 - p$$

0.2

$$E(X) = \sum x_i P(X = x_i) = p + (-1)(1 - p) = 2p - 1$$

0.3

$$p = \frac{E(X)}{2} + 1$$

0.4

$$Y = g(x) = X^2 = 1$$

sia per $X=1$ che per $X = -1$

0.5

$$E(Y) = E(g(X)) = E(X^2) = \sum g(X_i)P(X = X_i) = p + 1 - p = 1$$

0.6

$$\begin{aligned} \text{Var}(X) &= E(X^2) - E(X)^2 = E(Y) - E(X)^2 = 1 - (2p - 1)^2 = 1 - (4p^2 + 1 - 4p) \\ &= 4p - 4p^2 = \end{aligned}$$

$$4p(1 - p)$$

0.7

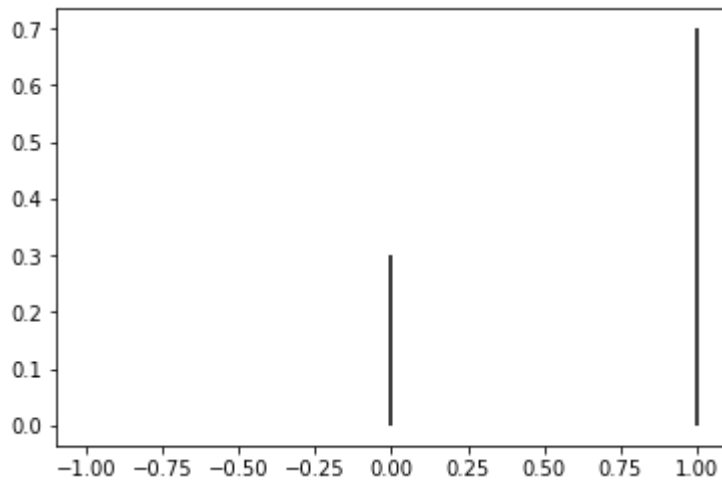
$h : \mathbb{R} \rightarrow \mathbb{R}$ tale che sia una bernoulliana.

$$Z = h(X) = \frac{X + 1}{2}$$

0.8

In [2]:

```
# grafico  $f_x, F_x, f_z, F_z$   
y = st.bernoulli(0.7)  
x = np.arange(-1,2)  
plt.vlines(x,0,y.pmf(x))  
plt.show()
```



Esercizio 1

1.1

$$E(\bar{X}) = E(X) = 2p - 1$$

1.2-1.3

$$Var(\bar{X}) = \frac{1}{n} Var(X) = \frac{4p(1-p)}{n}$$

2

$T_n = \frac{1+\bar{X}}{2}$ dimostrare che non è distorto per p

$$\begin{aligned} E\left(\frac{1+\bar{X}}{2}\right) &= E\left(\frac{1}{2} + \frac{1}{2} \sum \frac{X_i}{n}\right) = \frac{1}{2} + \frac{1}{2n} \sum E(X_i) = \frac{1}{2} + \frac{n}{2n} E(X) = \frac{1}{2} \\ &\quad + \frac{2p-1}{2} = \frac{2p-1+1}{2} = p \end{aligned}$$

3

$$P(|T_n - p| \leq 0.05)$$

Standardizzo

$$\begin{aligned} P\left(\frac{|T_n - 2p\sqrt{n}|}{\sigma} \leq \frac{2 * 0.05\sqrt{n}}{\sigma}\right) &= P(|Z| \leq \frac{0.1\sqrt{n}}{\sigma}) \approx \Phi\left(\frac{0.1\sqrt{n}}{\sigma}\right) - \Phi\left(-\frac{0.1\sqrt{n}}{\sigma}\right) \\ &= 2\Phi\left(\frac{0.1\sqrt{n}}{\sigma}\right) - 1 \end{aligned}$$

Esercizio 2

$$G \sim Exp(\nu)$$

2.1

$$D_X = [0, \infty)$$

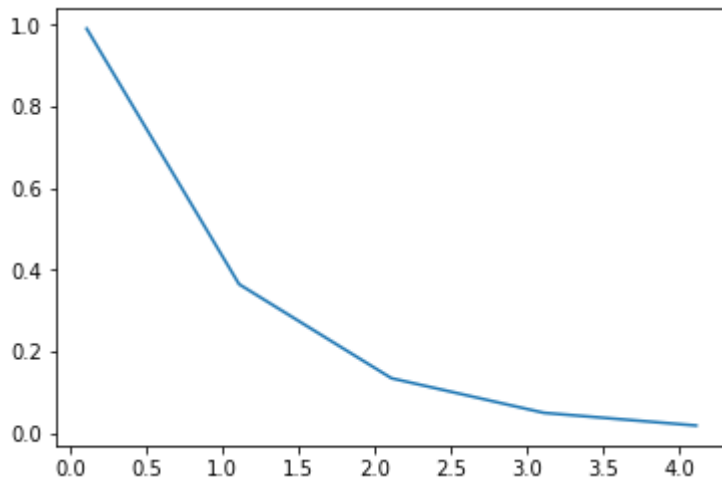
2.2

$$f_G = \nu e^{-\nu x}$$

2.3

In [3]:

```
nu = 0.1
y = st.expon(nu)
x = np.arange(y.ppf(0.01),y.ppf(0.99))
plt.plot(x,y.pdf(x))
plt.show()
```



2.4

$$\sqrt{\text{Var}(G)} = \frac{1}{\nu} = E(X)$$

2.5

$$E(G) = 10$$

figura a poichè l'area sopra la curva è maggiore.

In [4]:

```
car = pd.read_csv("carsharing.csv",delimiter=";",decimal=",")
car.columns
```

Out[4]:

```
Index(['CarIdentifier', 'TimeFrame', 'RushHour', 'PremiumCustomer', 'Distance',
      'Time'],
      dtype='object')
```

Esercizio 3

3.1

In [25]:

```
len(car)
```

Out[25]:

392

3.2.1

In [6]:

```
print("Qualitativo ORDINALE: {}".format(car['TimeFrame'].unique()))
```

Qualitativo ORDINALE: ['FRAME D' 'FRAME B' 'FRAME C' 'FRAME E' 'FRAME A']

3.2.2

In [7]:

```
len(car['TimeFrame'].unique())
```

Out[7]:

5

3.2.3

In [8]:

```
car['TimeFrame'].value_counts().sort_values().tail(2)
#pd.crosstab(index=car['TimeFrame'].sort_values(), columns=['Abs. Freq.'], colnames=[''])
```

Out[8]:

```
FRAME C    107
FRAME B    123
Name: TimeFrame, dtype: int64
```

3.2.4

In [9]:

```
pd.crosstab(index=car['TimeFrame'], columns=car['RushHour'], colnames=['Rush Hour'])
```

Out[9]:

Rush Hour	0	1
TimeFrame		
FRAME A	47	0
FRAME B	0	123
FRAME C	107	0
FRAME D	0	94
FRAME E	21	0

3.2.5

In [10]:

```
print("FRAME B e poi FRAME D")
```

FRAME B e poi FRAME D

3.3.1

In [26]:

```
carP = car[car['PremiumCustomer'] == 1]  
len(carP)
```

Out[26]:

227

3.3.2

In [27]:

```
carP['Distance'].mean()
```

Out[27]:

8.437444933920705

3.3.3

In [28]:

```
car.PremiumCustomer.mean()
```

Out[28]:

0.15816326530612246

3.3.4

Media campionaria

3.3.5

$$P(|Tn - E(X)| < 0.05) < 1 - \frac{Var(X)}{n * (0.05)^2}$$

Non so

In [31]:

```
1-(car.PremiumCustomer.std()/((0.05**2)*len(car.PremiumCustomer)))
```

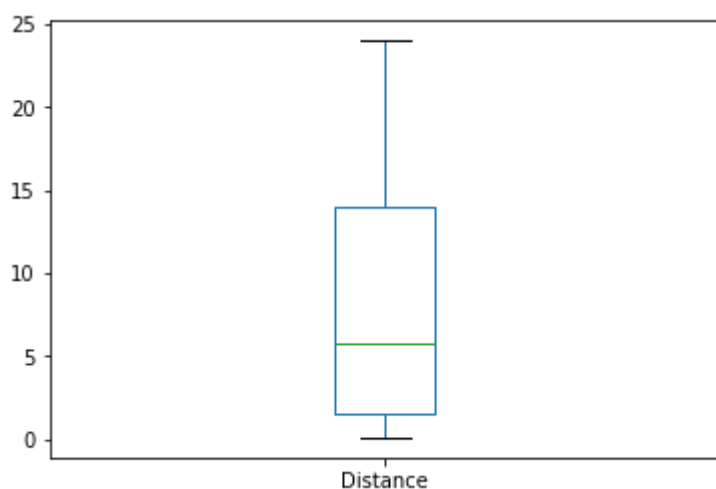
Out[31]:

-0.00885188191255537

3.4.1

In [15]:

```
car['Distance'].plot.box()  
plt.show()
```



3.4.2

In [16]:

```
car['Distance'].describe()
```

Out[16]:

```
count    392.000000  
mean       7.858673  
std        6.805123  
min        0.100000  
25%        1.575000  
50%        5.750000  
75%       14.025000  
max       24.000000  
Name: Distance, dtype: float64
```

In [17]:

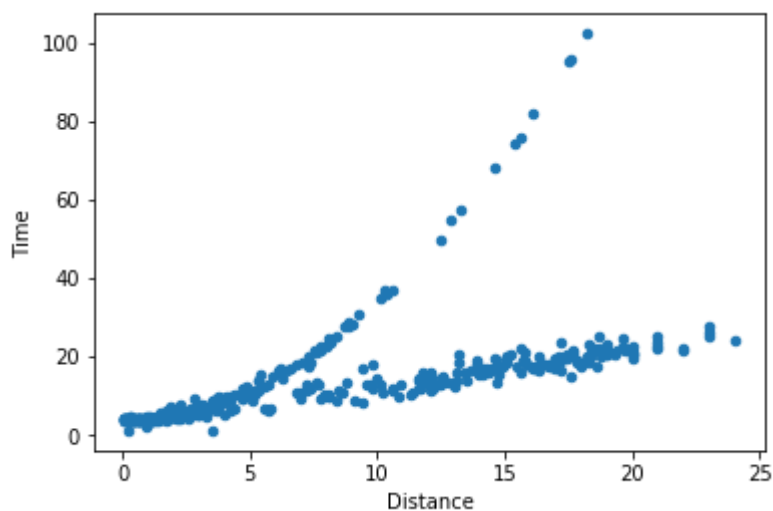
```
print('Indice di Centralità = Mediana: {} \nIndice di Dispersione = Range Interquartile  
: {}'.format(car['Distance'].quantile(0.5),(car['Distance'].quantile(0.75)-car['Distance'].quantile(0.25))))
```

```
Indice di Centralità = Mediana: 5.75  
Indice di Dispersione = Range Interquartile : 12.45
```

3.4.3

In [18]:

```
car.plot.scatter('Distance','Time')
plt.show()
print("Due Andamenti differenti. No relazione")
```



Due Andamenti differenti. No relazione

3.4.2

In [19]:

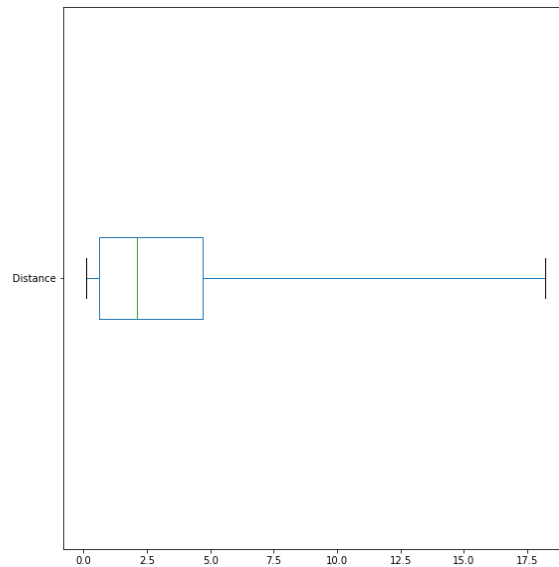
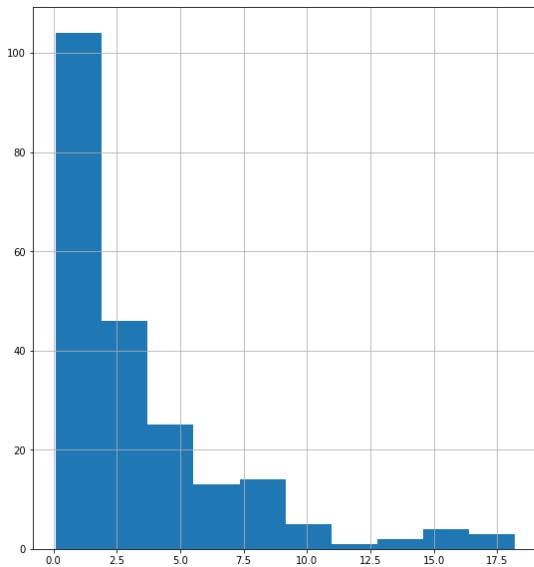
```
print("L'indice di correlazione {} conferma il fatto che non vi è una relazine di alcun  
tipo".format(car['Distance'].corr(car['Time'])))
```

L'indice di correlazione 0.6273992247694647 conferma il fatto che non vi è una relazine di alcun tipo

3.5.1

In [20]:

```
carD = car[car['RushHour'] == 1]['Distance']
plt.figure(figsize=[20,10])
plt.subplot(1,2,1)
carD.hist()
plt.subplot(1,2,2)
carD.plot.box(vert=False,whis='range')
plt.show()
```



3.5.2

In [21]:

```
print("No in quanto l'istogramma mostra che segueuna distribuzione esponenziale")
```

No in quanto l'istogramma mostra che segueuna distribuzione esponenziale

3.5.3

In [22]:

```
print(carD.mean())
print(carD.std())
```

3.3193548387096796
3.711106147915895

3.5.4

In [23]:

```
print("Esponenziale")
```

Esponenziale

3.5.5

In [24]:

```
print("Si perchè sono molto simili")
```

Si perchè sono molto simili